Card Trick No 19 – The 27 Card Trick – Corrected

10 July 2019

This document was developed by the author to correct Card Trick 19 in his book *The Hidden Paw's First 20 Logical Card Tricks*. The eBook is available with most online stores. (Click Here). All the book's details are presented in the author's website where this document was downloaded from:

http://thehiddenpaw.com/puzzle-books/the-hidden-paws-first-20-logical-card-tricks.html

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This trick comes from an earlier book by Martin Gardner. It is a bit elaborate arithmetically as it uses base 3 (or ternary) numbers to handle the collection of piles. It was described in his book: *Mathematics, Magic and Mystery* (Dover, 1956). There is a long discussion about this peculiar card trick in Chapter 3: "From Gergonne to Gargantua."

The approach is to give Victoria a random pile of 27 cards. She will select a card, which she has to remember and return to a random position within the remaining 26 cards. She will then think of a number N between 1 and 27 (inclusive) and let you know what it is.

The aim of the trick is to deal the pile 3 times into 3 piles of 9 cards each. After each deal, Victoria will point out the column or pile in which her card is. You will then collect the cards using a procedure presented below. After the third deal, you can show Victoria that her card is in the Nth position of the whole pile.

The trick depends on two procedures:

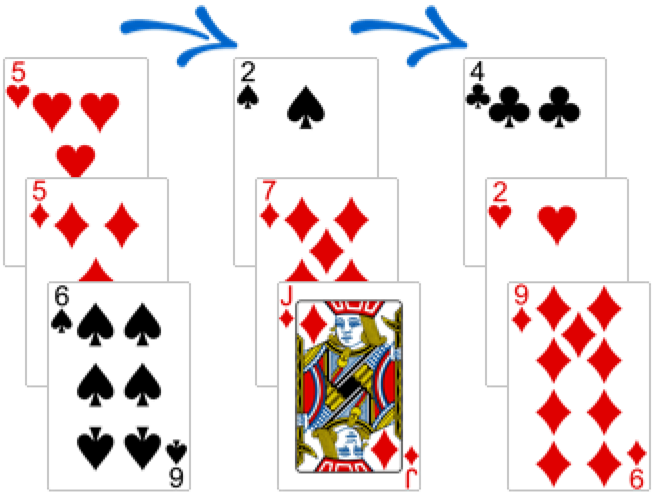
- To convert N – 1 into a base 3 (ternary) number (digits 0, 1 or 2)
- To deal the 27 cards into 3 piles of 9 each and collect them according to a procedure based on the ternary value of N - 1

1) Select 27 random cards from a deck. Let Victoria view them and select a card. She should remember the card then place it back into the pile of 26 in a position of her choice. You will not know what the card is nor its position.

2) Let Victoria select a number N between 1 and 27 (inclusive). She should tell what that number is.

3) Hold the pile of 27 cards in your hand, face down. Deal the cards down onto the table, face up and one by one. Start from your left and deal Card 1 onto the table, face up. Deal card 2 to its right and card 3 to the extreme right. Repeat this with cards 4, 5 and 6 and so on until you have 3 columns of 7 cards each.

This image shows the first 3 rows of the deal:



4) Ask Victoria to point to the pile she saw her selected card go into. Let us call this Pile P.

5) Collect the 3 piles according to the rules presented below and using the ternary number value of N – 1. As you collect them, pile them in your hand, face down.

6) After the first collection, deal the 27 cards again as you did in step 2. When Victoria points to the pile containing her card, again, use the rules presented below to collect the 3 piles.

7) Deal the cards a third time as in step 3. Again, ask Victoria to point out the column in which she saw her chosen card. Collect the 3 piles as per the procedure below.

8) Finally, deal the cards a last time as in step 3. Ask Victoria to point out the column in which she saw her chosen card. Again, use the rules presented below to collect the 3 piles.

With the collected cards facing downwards, deal down the cards face up on the tabletop. After you deal N – 1 cards, Victoria's selected card would marvelously appear in the Nth position from the top.

How to Convert Victoria's (N – 1) from the Decimal to the Ternary base:

Let us say Victoria select N = 22. This means that in the last distribution (the third), he card should be in the 22^{nd} position from the top. The trick relies on expressing N – 1 = 21 in the ternary base.

1) In the decimal system, we are allowed 10 single digits: 0 to 9. If we have a 4 digit number ABCD, we can express it as follows:

$$A \times 1000 + B \times 100 + C \times 10 + D \times 1$$

A, B, C and D can be any single digit from 0 to 9. Victoria's number is 2 x 10 +1 x 1. So, C = 2 and D = 1. The rest are zeros.

Numbers in any base b can be expressed as

$$A \times b^{(b)} + B \times b^{(b-1)} + C \times b^{(b-2)} \ldots\ldots G \times b^{(1)} + H \times b^{0}$$

Note each of the coefficients A, B, C, etc. cannot be larger than b-1. This is the same as with decimal numbers, where the largest digit is 9.

2) In the ternary system, we are allowed 3 single digits: 0, 1 and 2. For example, we can express a 4 ternary digit number ABCD as multiples of 3 as follows:

$$A \times 27 + B \times 9 + C \times 3 + D \times 1$$

For example, a ternary number such as 2201 = 2 x 27 + 2 x 9 + 0 x 3 + 1 x 1 = 54+18+1 = 73 in the decimal system.

To get the value of 21 in ternary, we need 3 ternary digits (since 26 in ternary is 222, our largest N-1).

So our ternary number where N < or = 26 has 3 digits: A x 9 + B x 3 + C x 1.

Find A: our first position from the left is the 9's. We know that 18 is the nearest number to Victoria's number, 21, so we choose A = 2. It cannot be 3 since that we be 27 which is the value of the 4th ternary position.

Find B: another 3 would raise the 18 to 21 so we have one 3 or B = 1.

Find C: since we have completed 21, the 1's digit should have a coefficient of 0, therefore C = 0.

So 21 (decimal) in ternary format = 210 = 2 x 9 + 1 x 3 + 0 x 1.

All we have to remember is the ternary value of Victoria's number: 2-1-0. It will indicate how we collect the piles.

How to collect the 3 piles after each deal:

The mathematical procedure requires us to **reverse the ternary number that represents N - 1**. So, we will now reverse 2-1-0 to become 0-1-2 (you need to retain the zeros).

- After deal 1, we use the first digit to the left (0 in our case)
- After deal 2, we use the second digit from the left (1 in our case)
- After deal 3, we use the third digit from the left (2 in our case)

As you remember, after each deal, you know which pile contains Victoria's chosen card. Call that pile P. No matter how you pick up the 3 piles, pile P should be placed in the position specified by these rules:

> a) If the digit for any deal = 2, place pile P face down in the palm of your hand. Then place the other 2 piles above it, in whichever order you wish.
> b) If the digit for any deal = 1, place pile P face down but between the other two piles: one above and one below (it does not matter which).
> c) If the digit for any deal = 0, place all other piles face down in your palm and place pile P above them.

It would be easier for you to remember letters for the above: (B)ottom, (M)iddle and (T)op.

Hint: you need to fool around when collecting the piles so that Victoria does not discover that you are following a specific order after each deal. Sometimes, pick up her pile first. Sometimes, pick up the rest and then slip pile P above or below. Try to make the collection of piles look random.

The first collection after deal 1: in our case, the first digit from the left = 0. According to the rules above, Victoria's pile should be placed at the top of the piles in your palm, face down. No re-deal.

The second collection after deal 2: we have 1 as the second digit from the left. Pile P should be in the middle

The third collection after deal 3: we have 2 as our 3's coefficient. Pile P should be at the bottom of the other piles.

So our placements during collection will be in the order TMB. Miraculously, Victoria's card will now be in the Nth position. Deal down N – 1 cards and then show her the chosen card.

Other examples:

1) If Victoria chooses N = 1, the ternary for N – 1 will be 000 so all 3 collections will force pile P to be on top of the others or TTT.

2) If she chooses N = 27, the ternary for N – 1 will be 222 so all 3 collections will force pile P to be at the bottom of the others or BBB.

3) Take any number N such as 12. N – 1 = 11 and in ternary, this is 102. Reverse it to become 201 which means our placements will be BTM.

4) If Victoria selects 14, our ternary for 14-1 = 13 will be 111 so we will be in the case of MMM.

Explanation of the Trick:

The best explanation is given by Matt Parker of Numberphile:

https://www.youtube.com/watch?v=l7lP9y7Bb5g&t=517s

Extensions of the Trick:

Matt Parker shows the same trick with 49 cards distributed over 7 piles of 7 cards each with two collections (using base 7). No explanation is given.

https://www.youtube.com/watch?v=G_OuIVOGDr8&t=34s

I will be developing a full explanation based on any base and any number of cards. Watch this space.